



The economics of stop-and-go epidemic control

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ABSTRACT

We analyse 'stop-and-go' containment policies that produce infection cycles as periods of tight lockdowns are followed by periods of falling infection rates. The subsequent relaxation of containment measures allows cases to increase again until another lockdown is imposed and the cycle repeats. The policies followed by several European countries during the Covid-19 pandemic seem to fit this pattern. We show that 'stop-and-go' should lead to lower medical costs than keeping infections at the midpoint between the highs and lows produced by 'stop-and-go'. Increasing the upper and reducing the lower limits of a stop-and-go policy by the same amount would lower the average medical load. But increasing the upper and lowering the lower limit while keeping the geometric average constant would have the opposite effect. We also show that with economic costs proportional to containment, any path that brings infections back to the original level (technically a closed cycle) has the same overall economic cost.

1. Introduction

As governments grappled with the second, third and fourth Covid-19 wave in Europe, they usually took a far more gradual and graduated approach than during the initial phase of the pandemic. At that time the number of seriously ill increased so rapidly that it overwhelmed health systems, in particular hospital capacities, in several countries. The second wave, which started with the onset of the flu season in the autumn of 2020, did lead to a somewhat moderated challenge for health systems.

After the first peak, the urgency to 'flatten the curve' [1] did subside to a certain extent. However, governments still felt the need to take measures to slow down the spread of the virus when the medical load was high. One key strategic issue facing authorities is whether they should try to preserve a status quo, or alternate lockdowns with periods of easing.

In Italy and England, the central governments instituted a tiered system with different levels of social distancing restrictions. In regions with a higher incidence of Covid-19, the restrictions are tighter. Within such a 'traffic light' system a region (city or other subdivision) can graduate to a lower level of restrictions if its epidemiological parameters improve, and, vice versa, restrictions will be tightened if cases increase again. These countries thus adopted de facto a 'stop-and-go' policy at the regional level.

A change into a higher or lower category will of course become more

frequent the closer the parameters defining the various tiers are. One key issue for this 'regional traffic light' approach is thus how wide apart these parameters should be set. We investigate this issue keeping in mind that social distancing measures have an economic cost, which increases with their severity. The choice of parameters should be informed by their economic cost, relative to the health benefits in terms of lower infections, hospitalisations and deaths [2].

We do not consider a general optimal control problem. Our aim is limited to comparing policies that make intuitive sense and that describe the choices of different European countries. At the national level one can observe that Germany's curve remained relatively flat compared those of France, Belgium or Spain. See Fig. 1.

There is one simple economic argument that would favour the 'stop-and-go' strategy of alternating harsh restrictions with broad easing. The economic cost of closing restaurants, closing schools or imposing restrictions on movement is the same whether the current rate of infections is high or low. This implies that one should use harsh restrictions when the case count is high because one would then achieve the largest fall in cases (in absolute numbers).

The argument against the 'stop-and-go' strategy is that the cost of the harsh restrictions to achieve a rapid fall in infection is likely to be convex. A small proportional reduction in infections (or rather the reproduction number) can be achieved by measures which have little impact on the economy (e.g. mask wearing, etc.). Achieving a swifter

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deceleration in the diffusion of the virus requires substantially stronger restrictions of the type mentioned above.

One could of course argue that stop-and-go policies are inferior to the ‘East Asian’ option of eradicating the virus [3], which then allows a total reopening of the economy. But this option had been abandoned in Europe as the draconian measures, including border closures that would be required, have apparently been widely judged as unacceptable. Note, however, that it is currently yet to be settled whether additional factors, like evolutionary adaptations, contribute to differences in the path the spread of the SARS-CoV-2 pathogen took in European and East-Asian populations [4].

The remainder of our contribution is organised as follows: we start by briefly reviewing the standard SIR model to which we add a relationship that describes the economic cost of reducing the spread of the virus. This framework is then used to examine the economic control costs of two alternative policies: keeping the medical load constant [5], versus a stop-and-go policy. We then compute the medical load implied by these two policies over a given time path and compare the resulting relative economic costs against the benefits in terms of a lower overall number of infected. Finally, we consider the implications of a time-varying native reproduction factor, for example an increase due to colder weather, leading to more indoor interactions.

Throughout, our purpose is not to describe and solve a general optimal control problem, but to compare the economic costs of different concrete policy options. Fig. 2 illustrates schematically the ‘stop-and-go’ epidemic control which we model below.

2. Modelling framework

We start with a short presentation of the standard SIR model where we denote with $S = S(t)$ the fraction of susceptible (non-affected) people, with $I = I(t)$ the fraction of the population that is currently ill (active cases, which are also infectious), and with $R = R(t)$ the fraction of recovered. Normalization demands $S + I + R = 1$ at all times. We write the continuous-time SIR model as

$$\tau \dot{S} = -gSI, \quad \tau \dot{I} = (gS - 1)I, \quad \tau \dot{R} = I, \quad (1)$$

which makes clear that τ is a characteristic time scale. Normalization is conserved, as $\dot{S} + \dot{I} + \dot{R} = 0$. Infection and recovery rates are g/τ and $1/\tau$. The number of infected grows as long as $\dot{I} > 0$, namely when $gS > 1$. Herd immunity is consequently attained when the fraction of yet unaffected people dropped to $S = 1/g$. The total number of past and present

infected is $X = 1 - S = I + R$.

From (1) one sees that g/τ governs the transition between two compartments, from susceptible to infected. This transition rate is constant within the basic version of the SIR model. There are two venues to relax this condition:

- **Non-linear reproduction rates.** The basic reproduction factor g may depend functionally on the actual number of infected I [7,8], or on the total number X [2]. This happens when societies react on an epidemic outbreak.
- **Time-dependent reproduction rates.** From the viewpoint of the pathogen, certain changes in the transmission rate $g = g(t)$ are external, e.g. because hosts decide more often to quarantine.

Here we focus on time-dependent $g = g(t)$, mostly as induced by stop-and-go politics, as illustrated in Fig. 2. We assume a stop-and-go cycle which repeats after $T = T_{up} + T_{down}$:

$$g(t) = \begin{cases} g_{up} > 1 & t \in [0, T_{up}] \\ g_{down} < 1 & t \in [T_{up}, T] \end{cases}. \quad (2)$$

The time spans during which epidemics expands/contracts are respectively T_{up} and T_{down} . The policy is cyclic if $I(0) = I(T)$, viz when the starting case number is reached again.

2.1. Low incidence approximation

We assume that infection counts are substantially lower than the population, viz that $I \ll 1$. For example, even in a highly affected country like Italy, the total number of daily infections has rarely exceeded thirty thousand [9], which corresponds to less than 0.0005 of total population. The total number of cases has reached 1.5 million, which is equivalent to $S \approx 0.975$. Given that the economic costs associated with raising infection numbers are based on estimates, as discussed in the subsequent section, the low incidence approximation provides in comparison sufficient accuracy.

Within the low-incidence approximation the number of susceptibles remains constant and we can set, without loss of generality $S \rightarrow 1$. Hence we need to deal only with

$$\tau \dot{I} = (g - 1)I, \quad I(t) = I(t_0) e^{(t-t_0)(g-1)/\tau}. \quad (3)$$

In the quasi-stationary state we have simple exponential growth/decay. For stop-and-go control the reproduction factor is piece-wise

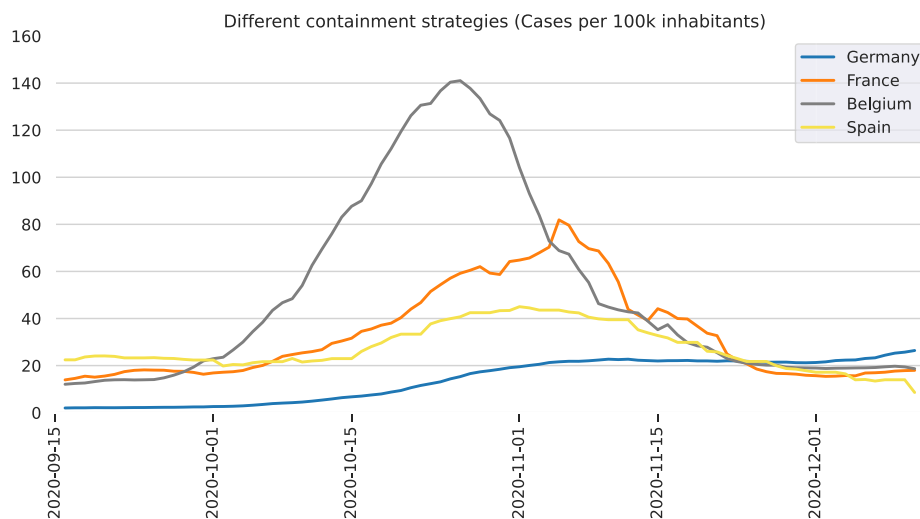


Fig. 1. Examples of second-wave control strategies. For the control of the second Covid-19 wave in Europe, most countries imposed comparatively strict lockdowns. Illustrative examples are, as shown, France, Belgium and Spain. As an exception, Germany, starting around the beginning of November 2020 opted for a ‘semi-lockdown’, which resulted in near constant infections. Graphic generated using the Goethe Interactive Covid-19 Analyser [6].

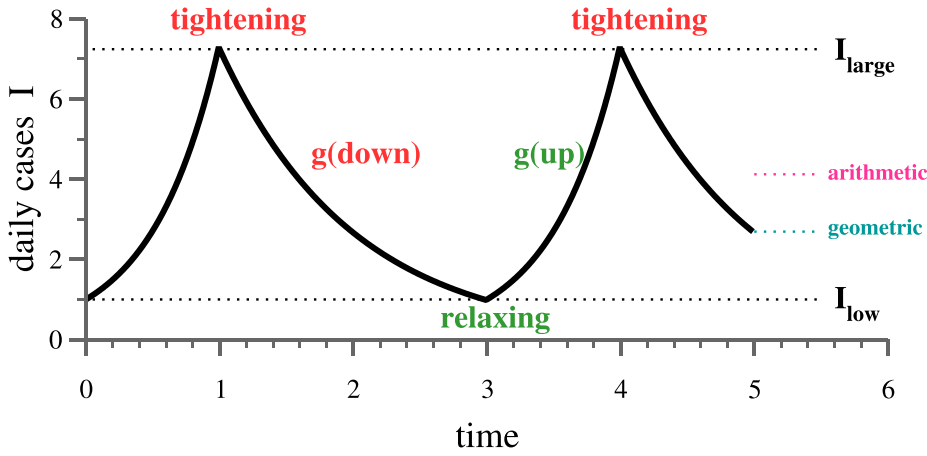


Fig. 2. Stop-and-go epidemic control. Containment policies that alternate periodically between I_{low} and I_{large} . When relaxing, the number of daily cases $I = I(t)$ expands at a rate $g(up) \equiv g_{up} > 1$. Policies are tightened again when case number are too high. Daily case numbers then contract with a rate $g(down) \equiv g_{down} < 1$. In the example shown, up- and down times are $T_{up} = 1$ and $T_{down} = 2$. For a presumed time scale of one month the period $T_{up} + T_{down}$ of the control cycle would be here three months. Also indicated are the arithmetic and the geometric means of I_{low} and I_{large} , as used in Sect. 4.1. Note that constant control is recovered in the limit $I_{low} \rightarrow I_{large}$.

constant, which allows us to evaluate explicitly the time evolution of case numbers, and with this the associated economic costs.

3. Economic costs of disease control

The economic costs of imposing social distancing on a wider population, closing restaurants or retail trade, are at the core of policy discussions. The key issue here is how these costs vary with the social distancing measures (so-called Non-Pharmacological Interventions, NPI) imposed. They increase in severity from mask requirements, abstaining from travel or restaurant meals to more invasive interventions like closure of schools, lockdowns or curfews. Limiting social interaction necessarily reduces economic activity. This suggests that the economic cost of the social distancing, measured as the proportional loss of GDP, should increase with the reduction in the transmission rate described by g ,

$$E = c_e \frac{g_0 - g}{g_0}, \tag{4}$$

where g_0 is the native reproduction factor. Here we assumed that social distancing costs are proportional to the percentage-wise reduction of the reproduction factor, viz to $(g_0 - g)/g_0$.

Our basic assumption, that social distancing costs are proportional to the reduction in the reproduction parameter, differs from the assumptions underlying matching models, as used in Ref. [10] or [11], which typically arrive at a quadratic relationship between the economic cost and the reduction in contagion, whereas [12] postulates simply a convex cost curve.

A concrete example can illustrate the key mechanism behind the matching framework, which usually assumes that the 'lockdown' takes the form of the confinement of a proportion of the population, and that contagion is possible only outside. If one half of the population has to stay at home, only the other half can go out and get potentially infected. But the half which is not confined will find only one half of their potential partners outside, resulting in one fourth of the number of matches. Contagion should thus be reduced by a factor of four when confining one half of the population.

In the matching framework, the economic cost is assumed to be proportional to the percentage of the population which is confined - not the number of matches. This framework thus separates social activity (matches, meetings of people) from economic activity, assuming that contagion is fostered only through social activity. Another implicit assumption behind this view is that those who are not confined will not have longer meetings with the ones they still find outside; or that those who are free to move accept to have only half of matches, and do not decide to meet somebody else if their preferred match is not available. These implicit assumptions are crucial. For example, contagion would

only be halved if those not confined would meet twice as long with the remaining matches they find. Some authors, e.g. Ref. [11] acknowledge these considerations by allowing for different economies of scale in matching.

Our view of lockdown or rather social distancing is that governments mandate the closure of some part of the economy, in reality mainly the services sector (restaurants, bars, shops, etc.). This is different from a strict confinement of a part of the population. A restaurant which is closed (or limited in its opening hours) results in less value added created and diminishes at the same time the potential for contagion. But the restaurant owners and their workers are not confined, they can meet others. There is thus no quadratic effect in terms of contacts. We thus start from the assumption that economic activity involves occasions for contagion, implying that the loss of economic activity should be directly proportional to the reduction in occasions for contagion and thus the effective reproductions rate. For related approaches see Refs. [2,13–15].

This view of 'lockdown' corresponds closer to the measures adopted by many governments during this second wave. The matching model might have been more appropriate during the first wave when indeed in some countries large parts of the entire population were forbidden to leave their home, except for essential business.

Finally we note that social distancing measures (NPIs) cannot affect the number of infected, only the rate at which their number grows over time. Eq. (4) implies that the only way to avoid all contagion is to completely shut down the economy. The parameter c_e represents a scaling factor, which depends on the structure of the economy (importance of services requiring close contact, like tourism) and the degree to which the population effectively adheres to official restrictions. It has been estimated that c_e is of the order of 0.25 [2].

3.1. Uniform control

We first briefly examine the implications of a policy which keeps the number of infected [5] and thus the medical load constant. Such a policy is of course not optimal, but it serves as a useful benchmark for our more general results. It can be considered as the limiting case of the 'traffic light' system in which the difference in parameters between tiers or levels becomes vanishing small.

Formally, uniform control implies a constant fraction $I \rightarrow I_{const}$ of infected. This is achieved for $g = 1$, independent of the value of I_{const} . The economic cost E_{const} per time unit is therefore

$$E_{const} = c_e \frac{g_0 - g}{g_0} \Big|_{g=1} = c_e \frac{g_0 - 1}{g_0}, \tag{5}$$

Note that E_{const} is independent of the value of the medical load one wants to retain. As already mentioned above, the economic costs of keeping the reproduction factor at one is independent of how many

infected there are.

3.2. Stop-and-go control

Stop-and-go, or ‘bang bang’, control corresponds to an on-off policy as illustrated in Fig. 2 above, which can be described within the framework developed here by the following rule: Control is increased when $I = I(t)$ reaches an upper threshold I_{large} , and decreased when $I = I(t)$ falls below a lower threshold I_{low} .

We denote with $g_{\text{down}} < 1$ the small reproduction rate corresponding to strong control, and with $g_{\text{up}} > 1$ the large reproduction factor corresponding to weak control. Using the low-incidence approximation (3) we find

$$T_{\text{up}} = \frac{\tau}{g_{\text{up}} - 1} \ln\left(\frac{I_{\text{large}}}{I_{\text{low}}}\right), \quad T_{\text{down}} = \frac{\tau}{g_{\text{down}} - 1} \ln\left(\frac{I_{\text{low}}}{I_{\text{large}}}\right) \quad (6)$$

by integrating $I(t)$ from $t = 0$ to $t = T_{\text{up}}$, the time needed for $I(t)$ to grow from I_{low} to I_{large} , with a respective expression for the down-time T_{down} .

As one would expect from an exponentially growing variable, the time needed to evolve from one value to another is a function of the logarithm of the ratio of end- and starting points, being inverse to the effective growth rate of infections (held constant by restrictions). Given that the economic costs are constant per unit of time (lower during the period of allowing infections to increase, higher during the restrictive phase) one arrives at the following solution for the total economic costs:

$$E_{\text{bang}} = \frac{c_e}{g_0} [(g_0 - g_{\text{up}})T_{\text{up}} + (g_0 - g_{\text{down}})T_{\text{down}}] \frac{1}{T_{\text{up}} + T_{\text{down}}} \quad (7)$$

when considering an entire period or cycle, up and down when stop-and-go control is applied.

3.3. Vanishing cost differential

The cost difference between bang-bang and constant control is

$$E_{\text{bang}} - E_{\text{const}} = \frac{c_e}{g_0} \left[1 - \frac{g_{\text{up}}T_{\text{up}} + g_{\text{down}}T_{\text{down}}}{T_{\text{up}} + T_{\text{down}}} \right] \quad (8)$$

Note that $\ln(I_{\text{large}}/I_{\text{low}}) = -\ln(I_{\text{low}}/I_{\text{large}})$, which implies that both τ and $\ln(I_{\text{large}}/I_{\text{low}})$ drop out of (8), which vanishes as

$$E_{\text{bang}} - E_{\text{const}} = \frac{c_e}{g_0} \left[1 - \frac{g_{\text{up}}(g_{\text{down}} - 1) - g_{\text{down}}(g_{\text{up}} - 1)}{g_{\text{down}} - g_{\text{up}}} \right] \equiv 0. \quad (9)$$

This implies that both control types, constant and stop-and-go control, come with the same economic costs.

3.4. Neutrality theorem

The result, that the cost differential between constant and stop-and-go control vanishes, can be generalised if we rewrite (3) as

$$\tau \dot{I}_{\text{log}} = g - 1, \quad I_{\text{log}} = \ln(I), \quad (10)$$

where $g = g(t)$ is now an arbitrary function of time. We then have

$$I_{\text{log}}(t_{\text{end}}) - I_{\text{log}}(t_{\text{start}}) = \int_{t_{\text{start}}}^{t_{\text{end}}} \dot{I}_{\text{log}} dt = \int_{t_{\text{start}}}^{t_{\text{end}}} \frac{g(t) - 1}{\tau} dt, \quad (11)$$

which proves that

$$\frac{1}{t_{\text{end}} - t_{\text{start}}} \int_{t_{\text{start}}}^{t_{\text{end}}} g(t) dt = 1 \quad (12)$$

for closed trajectories, viz when $I_{\text{log}}(t_{\text{end}}) = I_{\text{log}}(t_{\text{start}})$. Comparing with (5) shows that average economic costs are independent of which timeline $g(t)$ is used for controlling the epidemic. Note that the case of constant control, $g(t) \equiv g$, is included as a special case. If the economic costs

of control are proportional to the reduction in the reproductions rate, one finds thus a ‘neutrality theorem’ for epidemic control. All trajectories which return to the point of departure (in terms of the infection rate) will lead to the same economic cost.

4. Mean number of infected under different control policies

The number of infected becomes the key criterion if the economic cost of different control policies (over complete cycles) is the same.

For constant infection rates g , the cumulative number of infected between two times $t = t_0$ and $t = t_1$ is

$$\begin{aligned} X_{0,1} &= \int_{t_0}^{t_1} I(t) dt = I(t_0) \int_{t_0}^{t_1} e^{(t-t_0)(g-1)/\tau} dt \\ &= \frac{I_0 \tau}{g-1} (e^{(t_1-t_0)(g-1)/\tau} - 1) = \frac{(I_1 - I_0) \tau}{g-1} \end{aligned} \quad (13)$$

when using (3) and that $I_1 = I_0 \exp((t_1 - t_0)(g - 1)/\tau)$. Noting that (6) holds generally for constant g , we have

$$\Delta T_{0,1} = t_1 - t_0 = \frac{\tau}{g-1} \ln\left(\frac{I_1}{I_0}\right), \quad (14)$$

which leads to

$$\frac{X_{0,1}}{\Delta T_{0,1}} = \frac{I_1 - I_0}{\ln(I_1/I_0)} \quad (15)$$

for the overall number of infected, on the average per time unit. Note that

$$\ln\left(\frac{I_1}{I_0}\right) = \ln\left(\frac{I_1 - I_0 + I_0}{I_0}\right) \approx \frac{I_1 - I_0}{I_0} \quad (16)$$

for $I_1 \approx I_0$, from which the limit

$$\lim_{I_0 \rightarrow I_1} \frac{X_{0,1}}{\Delta T_{0,1}} = I_0 \quad (17)$$

is recovered. Empirically it been observed that the cumulative medical load during the ‘down phase’ is about 30% higher than the cumulative load which follows the peak [16].

4.1. Bang-bang infection numbers

For stop-and-go control we have two periods with constant g , when I goes up and respectively down. With (15) we find that the total cumulative fraction of infected is determined by;

$$X_{\text{bang}} = \frac{I_{\text{large}} - I_{\text{low}}}{\ln(I_{\text{large}}/I_{\text{low}})} T_{\text{up}} + \frac{I_{\text{low}} - I_{\text{large}}}{\ln(I_{\text{low}}/I_{\text{large}})} T_{\text{down}} \quad (18)$$

for bang-bang control, and hence

$$\bar{I}_{\text{bang}} = \frac{X_{\text{bang}}}{T_{\text{up}} + T_{\text{down}}} = \frac{I_{\text{large}} - I_{\text{low}}}{\ln(I_{\text{large}}/I_{\text{low}})} \quad (19)$$

for the time-averaged number \bar{I}_{bang} of infections.

Here we are interested in on-the-average stationary control strategies, for which the level I of infections is kept at the average. In Sect. 3.4 we did show that stationary control results always in identical economic costs. This holds however not for the medical load, which can be considered to be the proportional to the average infection number \bar{I} . It is in particular of interest to compare the medical \bar{I}_{bang} , of stop-and-go control, with the policies keeping I at a constant, intermediate level or benchmark. A natural choice for this benchmark could be the arithmetic mean, or midpoint, $I_{\text{mid}} = (I_{\text{large}} + I_{\text{low}})/2$. However, we will consider as well the geometric mean $I_{\text{geo}} = \sqrt{I_{\text{large}} I_{\text{low}}}$.

For the arithmetic mean, the ratio $\bar{I}_{\text{bang}}/\bar{I}_{\text{mid}}$ in mean infection numbers is

$$\begin{aligned} \delta \bar{I} = \bar{I}_{\text{bang}}/\bar{I}_{\text{mid}} &= \frac{X_{\text{bang}}}{T_{\text{up}} + T_{\text{down}}} \frac{2}{I_{\text{large}} + I_{\text{low}}} \\ &= \frac{x-1}{\ln(x)} \frac{2}{x+1} \leq 1, \end{aligned} \tag{20}$$

when denoting $x = I_{\text{large}}/I_{\text{low}}$. In the limes $I_{\text{large}} \rightarrow I_{\text{low}}$, viz $x \rightarrow 1$, one has $\lim_{x \rightarrow 1} \delta \bar{I} = 1$, as expected. The limes $x \rightarrow 1$ is performed using the small $x - 1$ expansion $\ln(x) = \ln(1 + (x - 1)) \approx x - 1$.

That $\bar{I}_{\text{bang}}/\bar{I}_{\text{mid}}$ is strictly smaller than unity for $x > 1$ can be seen, e.g., by plotting (20) as a function of x , as done in Fig. 3. Numerically one finds a reduction of 29% at $x = 10$, which is somewhat higher than ratio of 7 to 1 observed in the actual values displayed in Fig. 1. Note that one has $2/(x + 1) \leq 1$ when $x \geq 1$, for the second term in (20), with $(x - 1)/\ln(x) \leq 1$ holding for the first term. The last relation holds because the log-function is concave, with a slope $d \ln(x)/dx = 1$ at $x = 1$.

The result that $\bar{I}_{\text{bang}} \leq \bar{I}_{\text{mid}}$ implies that a policy of stop-and-go is superior to (less bad than) a policy of keeping infections constant half-way between the peak and trough. The two policies would have the same economic cost, but stop-and-go would lead to a lower overall medical load. Neither policy would of course be optimal in an unconstrained policy space. But our aim is merely to consider policies that have been adopted. The intuition behind this result can be seen from Fig. 2 above. The infection curve lies more time below than above the arithmetic average, a well-known property of exponential growth.

The constant control policy serves only as benchmark. The results are much more general: As can be seen from (20) the average medical load falls as the ratio of the upper limit to the lower limit increases, while holding the (arithmetic) average constant. This implies that the medical load resulting from a 'stop-and-go' policy improves as one increases the upper and reduces the lower limit by the same amount.

Regarding the comparison to the geometric mean I_{geo} , we consider

$$\begin{aligned} \bar{I}_{\text{bang}}/\bar{I}_{\text{geo}} &= \frac{I_{\text{large}} - I_{\text{low}}}{\ln(I_{\text{large}}/I_{\text{low}})} \frac{1}{\sqrt{I_{\text{large}}I_{\text{low}}}} \\ &= \frac{x-1}{\ln(x)} \frac{1}{\sqrt{x}} \geq 1, \end{aligned} \tag{21}$$

where we used $x = I_{\text{large}}/I_{\text{low}}$, as for (20). The functional dependence is included in Fig. 3. It can be seen that it is favorable to keep the incidence at the geometric mean, instead of letting it oscillate between I_{low} and I_{large} . Moreover, the ratio of the respective average medical loads

increases only modestly as the ratio between upper and lower limit increases. Letting case number vary by a factor of ten leads to an increase of 24%. It should not be surprising that keeping infections at the geometric mean is worse, given that the distance between the two means increases as the ratio of the upper to the lower limit becomes larger. For $9 = I_{\text{large}}/I_{\text{low}}$, the arithmetic mean is equal to 5 whereas the geometric mean is 3. Choosing the geometric as the intermediate point thus amounts to choosing a higher benchmark.

From a general viewpoint, it follows from (21) that the average medical load increases as the ratio of the upper limit to the lower limit increases, when holding the geometric average constant. This implies that the medical load resulting from a 'stop-and-go' policy increases as one increases the upper and reduces the lower limit by the identical factor. These considerations suggest that the decision regarding how wide apart to set the limits of a 'stop-and-go' or a regional 'traffic light' policy depends on whether one wants to keep the arithmetic or the geometric average constant.

4.2. Significance

The results discussed above apply only within the limits of hospital capacity. Once that limit has been reached any further increase in the medical load would imply rapidly rising medical and ethical costs. A key assumption made above is that the medical load is proportional to the number of infections. This is a reasonable assumption as long as each infected can be cared for, even when severe symptoms requiring hospitalisation develop. Hospital capacity and especially the number of intensive care units (ICU) create thus a practical upper bound on political choices. The upper (and lower) limit cannot be increased beyond the range given by hospital capacity. The question we address is to what extent this range should be used by the authorities.

A lower medical load constitutes a sufficient criterion for preferring the stop-and-go policy, given that the respective economic costs vanish for different control paths as long as the initial infection incidence is reached again. Note that the medical load can be translated into economic costs [15,17–19].

Most of the literature focuses on the economic value of the lives lost. However, that might be a mistake [2], as infections with less severe symptoms can also lead to considerable economic costs. One example of the economic cost of infections would be the loss of working time of those infected and with mild symptoms, when these individual have to self-isolate and cannot work for a certain time. This loss could be calculated as the number of weeks of working time due to symptoms (and self-isolation needs) and would be equal to a proportion of GDP [2].

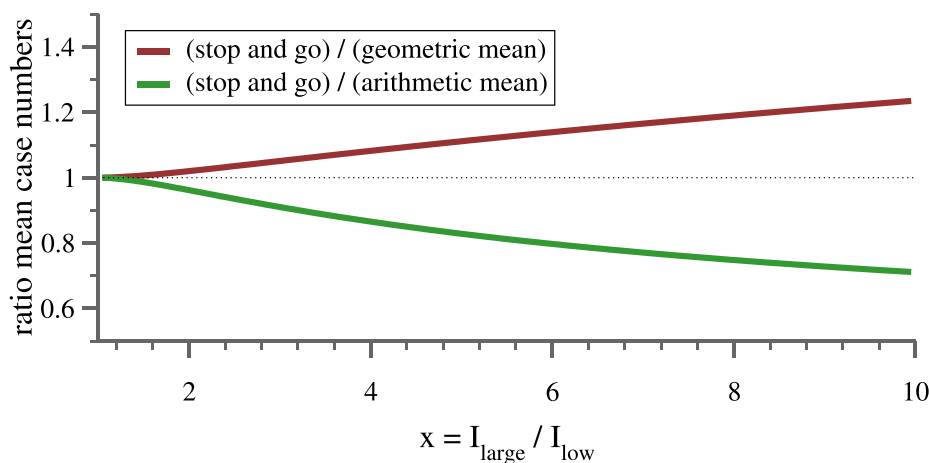


Fig. 3. Relative case numbers for distinct control policies. For stop-and-go control case numbers oscillate between I_{low} and I_{large} . Alternatively considered are constant incidences, either at the arithmetic mean, $(I_{\text{low}} + I_{\text{large}})/2$, or at the geometric mean, $\sqrt{I_{\text{low}}I_{\text{large}}}$. Shown are the ratios of the respective per time case numbers, as given by Eqs. (20) and (21).

To this one would have to add the hospitalisation costs for those with stronger symptoms and finally the economic value of lives lost. The lower medical load implied by a stop-and-go policy would thus also lead to lower overall economic costs.

A key difference between the present study and most existing literature concerns the kind of policies examined. Here we focus on a comparison between two representative, real-world policies, stop-and-go vs. constant control. Optimal control, which usually does not result in reversals of restrictions, is in contrast the goal for the majority of studies published so far. Contributions like [10,11] employ the matching framework, which is based on confinement as the main policy instrument and allows for economies of scale in lowering contagion as explained above. In such a framework it is clear that the optimal policy would be to impose tight restrictions from the start until the desired incidence is attained. For example [10], finds that “the optimal confinement policy is to impose a constant rate of lockdown until the suppression of the virus in the population.”.

Other contributions, which do not incorporate economies of scale in containment policies (either because they use constant returns in matching or because of a different view of social distancing restrictions) arrive at somewhat differentiated conclusions. For example [15], where containment works like a consumption tax, find that containment should build up gradually and peak early when there exists the perspective of a vaccine being discovered.

5. Changing epidemic parameters

The native reproduction factor g_0 is, as a matter of principle, an intrinsic property of the virus. As such it can be measured only at the very start of a pandemic, namely when nobody is yet aware of what is happening. However, this reproduction factor can change over time, for example with the season. It is well known that the danger of contagion is much higher indoors than outdoors. It is in fact nearly impossible to catch the Coronavirus outside [20]. In the case of influenza virus this change in the reproduction rate leads to the typical ‘flu season’, which starts with the onset of colder weather (late in the year in the Northern Hemisphere [21]). For the case of the Covid-19 virus other seasonal factors have also been mentioned, for example fluctuations in UV light [22,23], for a survey see Ref. [24].

One thus needs to consider time-dependent g_0 , which would correspond to the underlying spreading rate in a given societal state, i.e. without spontaneous behavioural modifications or explicit, government-imposed restrictions.

We assume that g_0 changes right at the top for the case of bang-bang control. Going up/down we then have $g_0 = g_0^{(up)}$ and $g_0 = g_0^{(down)}$. As a further simplification we postulate

$$T_{up} = T_{down}, \quad g_{up} - 1 = 1 - g_{down}, \tag{22}$$

which is equivalent to $g_{up} + g_{down} = 2$. Compare (6).

5.1. Costs for the economy

Given that we assume that $T_{up} = T_{down}$, the per time economic costs of keeping constant infection numbers is

$$E_{const} = \frac{c_e}{2} \left[\frac{g_0^{(up)} - 1}{g_0^{(up)}} + \frac{g_0^{(down)} - 1}{g_0^{(down)}} \right] \tag{23}$$

For bang-bang control we have instead

$$E_{bang} = \frac{c_e}{2} \left[\frac{g_0^{(up)} - g_{up}}{g_0^{(up)}} + \frac{g_0^{(down)} - g_{down}}{g_0^{(down)}} \right] \tag{24}$$

The difference is

$$E_{bang} - E_{const} = \frac{c_e}{2} \left[\frac{1 - g_{up}}{g_0^{(up)}} + \frac{1 - g_{down}}{g_0^{(down)}} \right], \tag{25}$$

which simplifies to

$$E_{bang} - E_{const} = \frac{c_e}{2} \left[\frac{1}{g_0^{(down)}} - \frac{1}{g_0^{(up)}} \right] \underbrace{(1 - g_{down})}_{>0} \tag{26}$$

when using the $T_{up} = T_{down}$ condition that $g_{up} - 1 = 1 - g_{down}$, see (22). Going down we restrict, viz $g_{down} < 1$ holds.

Bang-bang control is therefore favorable when $g_0^{(down)} > g_0^{(up)}$. This result would support the pattern observed in Europe where during the summer (when g_0 was lower than before) governments eased restrictions, imposing them again during the fall when g_0 increased again. The economic cost of the restrictions could thus be concentrated during the period when they had a higher yield in terms of infections avoided.

6. Conclusions

During the first wave of the Covid-19 pandemic the key concern was to ‘flatten the curve’. Harsh lockdown measures were needed when health systems were overwhelmed by the sudden increase in hospitalisations, many of which required intensive care units. The second wave resulted in a somewhat lower medical load, but it proved nevertheless indispensable to re-introduce some social distancing measures as otherwise the case load would have continued to increase at a near exponential rate. Countries have taken different approaches in this regard. In some, the measures have been just enough to stabilise infections. In others, the measures led to a strong fall in new cases and governments lifted restrictions - which in some cases led to a renewed increase.

Several countries introduced, interestingly, regionally graduated systems, which allow regions and cities to oscillate between periods of harsh restrictions that are triggered when infection numbers exceed certain thresholds, and periods of lower restrictions, which start when numbers have fallen again, now below a given threshold. These kind of quasi-automatised threshold containment policies are a graded realisation of the stop-and-go policy examined in the present study.

In the context of our model we have shown that any time path of restrictions which returns to the point of departure in terms of the infection rate, a scenario likely to occur within rule-based ‘traffic light’ systems, implies the same overall economic costs. Furthermore, our analysis suggests that stop-and-go policies might not be as costly as it could appear at first sight, at least if compared to the alternative of keeping the incidence rate constantly at the midpoint. The economic cost would be the same, but the overall medical load of stop-and-go should be lower. The result is reversed for the alternative of keeping infections at the (lower) geometric mean. Compare Fig. 2.

Applied to regional ‘traffic light’ systems, our results imply that increasing the upper, and reducing at the same time the lower threshold by the identical absolute amounts ΔI , via $I_{upper} \rightarrow I_{upper} + \Delta I$ and $I_{lower} \rightarrow I_{lower} - \Delta I$, leads to a lower medical load. The opposite would be true if one were to increase the upper, and at the same time reduce the lower threshold by the same proportion, $f_I > 1$, this time using the rescaling $I_{upper} \rightarrow I_{upper} f_I$ and $I_{lower} \rightarrow I_{lower} / f_I$. The latter procedure would increase the arithmetic average since the absolute increase in the upper limit would be now higher than the fall in the lower limit. This implies that any choice regarding the range between the upper and lower thresholds in a regional ‘traffic light’ must take this difference between arithmetic and geometric mean into account. In political reality the arithmetic average seems to be the more important benchmark as many comparisons across countries focus on the average number of infections over a given period.

We have purposely not formulated an optimal control problem because our aim was to provide a framework for studying policies that

have been widely adopted in the real world. However, the results concerning the arithmetic mean could also be interpreted as implying that the ‘optimal control’ (always within the class of policies we consider), would be to increase the range (viz ΔI) up to the maximum permitted by hospital capacity.

We also show that it makes sense for policies to react to seasonal variations in the native reproduction factor. The economic cost of tight social distancing should be incurred in winter, when infections would otherwise be high and rising.

Author statement

The two authors, Claudius Gros and Daniel Gros contributed equally to this article.

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